Monte Carlo Modelling in Visualyse Professional

Abstract: A range of methodologies can be used to analyse interference between radio communication systems, including static, area, dynamic and Monte Carlo. But what is Monte Carlo modelling and what are its advantages and disadvantages? This White Paper describes the Monte Carlo methodology and gives examples of its use with the Visualyse Professional study tool.

Static Analysis

The starting point to understand Monte Carlo analysis is the basis of most interference analysis: the static analysis.

Static analysis, as its name suggests, is a snapshot in which there is just the one value of each of the input parameters used to calculate interference. So there is one value for each of:

- Station locations: positions and heights
- Antenna pointing and gain patterns
- Propagation: suitable models together with, where applicable, a percentage of time and / or location
- Link attributes: transmit power, bandwidth, frequency
- Traffic (if used): link activation, traffic level etc.

From this single set of inputs, it is then possible to calculate the wanted and interfering link budgets for all the stations in the scenario.

This is a very useful type of analysis to gain an understanding of a problem but limited, as there can be so much variation in each of the input parameters.

A model with (say) 20 input variables each with 5 possible values results in around 9.5 x 10^{13} possible combinations. It is clearly not possible to examine all of them – so how to analyse scenarios with inputs that could take multiple values or vary over a range?

What is Monte Carlo Modelling?

Monte Carlo modelling is one approach to handle this variation. Where there are input parameters that could take multiple values or vary over a range they are sampled at random.

A Monte Carlo analysis involves taking multiple snap-shots or trials, each of which is a static analysis as described above. For each trial, the input parameters that vary are sampled at random, and the effect is to convolve their distributions together in the interference engine.

Examples of input parameters that could vary include:

- Selecting the position of a mobile system's user terminal (UT) at random over a base station sector
- Selecting the transmit power to use at random from a distribution
- Switching a link on or off, as specified by a traffic model
- Selecting a frequency from a set of alternatives (e.g. in a channel plan)
- Selecting a percentage of time and / or percentage of locations in a propagation model etc.

The results will depend on the distributions used for each of these input parameters so it is important that care be taken when selecting them.

Outputs

The outputs of a Monte Carlo analysis are statistical in nature. Typically there is a threshold value, so that for a metric X the system is degraded if:

$$X > T(X) \quad \text{or} \quad X < T(X)$$

The sign depends upon the metric X, so, for example, it differs when considering $X = I/N$ or $X = C/(N+I)$ as two possible metrics where the thresholds might be:

$$I/N \quad > T(I/N) = -10 \text{ dB}$$

$$C/(N+I) \quad < T(C/(N+I)) = 10.5 \text{ dB}$$

Then for each metric X with threshold $T(X)$ there will be two types of output:

1. Statistics $S[X]$, such as the probability that the threshold $T(X)$ is met
2. Cumulative distribution function (CDF) of X over the simulation

Note that it is not possible to derive information about interference event lengths as each trial is assumed to independent of the other.

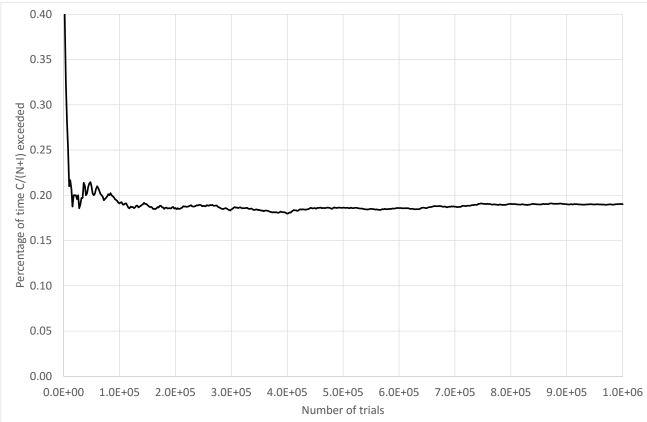
Testing for Validity

Monte Carlo modelling introduces a random element in so that outputs are statistical in nature. These results will alter as the number of trials increases, typically stabilizing after sufficient number of steps: but how many steps are required to achieve statistically valid results?

For example, in the study described below of mobile into digital terrestrial television (DTT) receive, the threshold was:

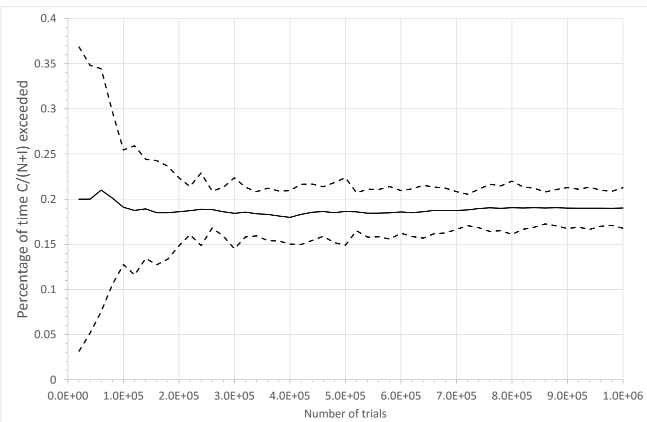
$$T(C/(N+I)) = 8 \text{ dB}$$

It is then possible to plot how the likelihood that threshold is exceeded varies by number of trials:



It can be seen that there is initially significant variation in this likelihood until the statistics stabilize. But how many trials are necessary?

One approach is to group the results and output the average over that group. It is then possible to calculate the standard deviation of the average and observe how this converges, as in the figure below:



This leads to a confidence interval derived by a specified number of standard deviations either side of the mean.

Why Undertake Monte Carlo Analysis?

Monte Carlo methodologies can appear daunting at first. It requires that decisions be made as to the distributions to use for the input parameters and care is required when interpreting results.

However, using Monte Carlo methods has many advantages, including:

- It is able to model variation of any number of inputs
- It can be applied to a wide range of sharing scenarios
- It generates statistics *S*[*X*] that can be used to answer key study questions.

In general, using Monte Carlo methods to analyse sharing scenarios will lead to more spectrum efficient solutions. It is also able to model more complex sharing scenarios compared to static methodologies such as the minimum coupling loss (MCL) approach.

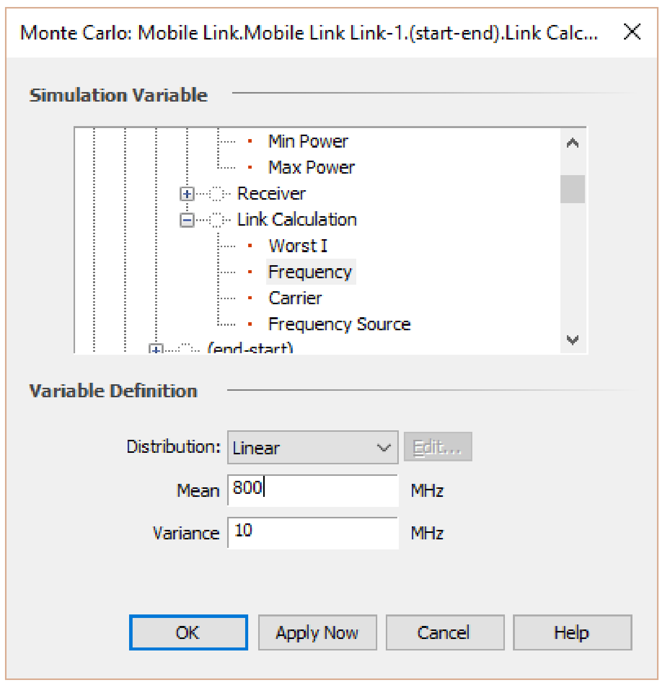
Visualyse Professional

Our desktop study tool Visualyse Professional can be used to analyse radio systems including both terrestrial and satellite networks. One of its strengths is it is able to model a wide range of methodologies including static, area, dynamic, Monte Carlo and area Monte Carlo.

The Monte Carlo methodologies use the features in the Define Variable and (in some cases) Traffic Modules. In particular, the Define Variable Module includes features to:

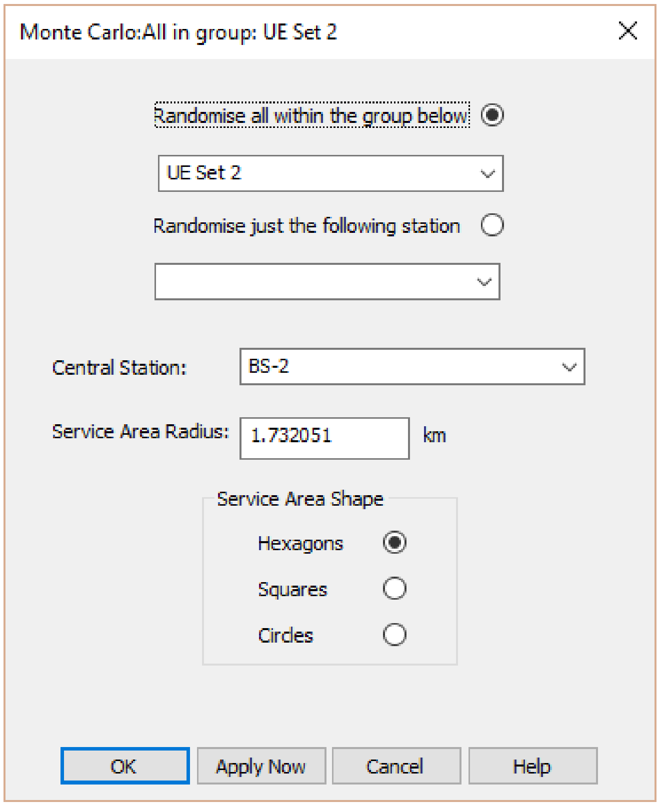
- Randomize any input parameter
- Randomize the position of terrestrial stations (e.g. mobile UTs) within an area, defined either as a circle, square or hexagon
- Randomize the positions of satellites in a constellation.

An example of the randomization of any input parameter is shown in the figure below:

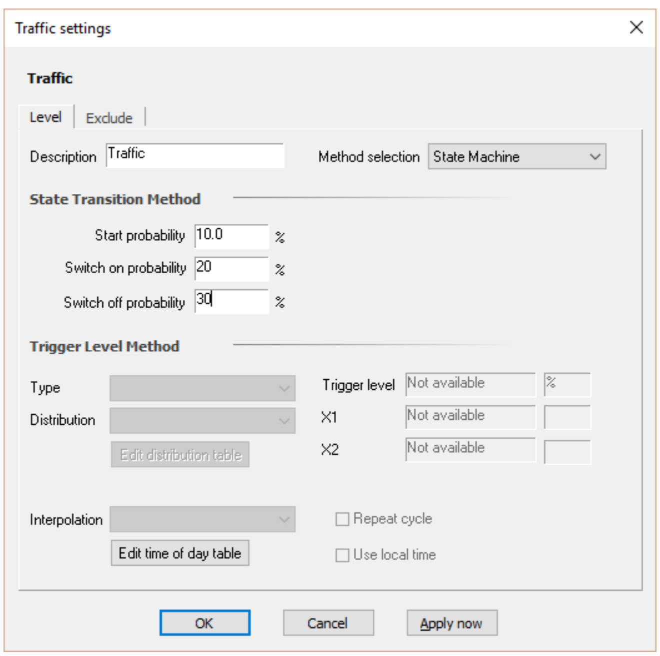


Distributions that can be modelled included linear (uniform), triangular, normal, Poisson, Rayleigh, Gamma, exponential and Erlang, plus the option to enter a user specified distribution

The dialog to randomize the positions of terrestrial stations within a service area is shown below:

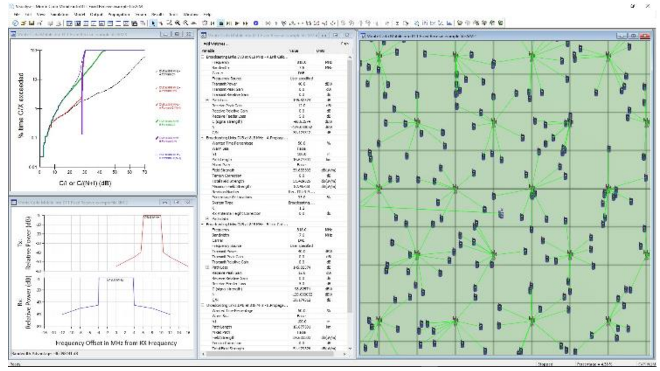


The traffic module allows links to be switched on and off using a state machine or time of day profile, as in the following dialog:



Example Analysis

The Visualyse Professional software includes several example files that show Monte Carlo methods in action. In particular, under Digital Dividend there is the file "Monte Carlo Mobile into DTT Fixed Receive.sim" as shown in the screen shot below:



In this simulation:

- The positions of the UTs have be randomized around each of the base stations. This impacts the transmit power via a power control loop
- The position of the DTT receiver has been randomized

The scenario is also non-co-frequency, so the interference is calculated by integrating the:

- Transmit spectrum mask
- Receive filter mask.

This can be seen in the frequency view.

The result are statistics on the C/I, C/N and C/ (N+I) for the two cases:

- Mobile downlink (base station transmit) into DTT receive case
- Mobile uplink (UT transmit) into DTT receive

Contact and further information

As well as Visualyse Professional, Transfinite have a range of other tools including:

- Visualyse GSO: to support the coordination of GSO satellite systems
- Visualyse EPFD: to analyse whether non-GSO systems meet the EPFD limits in Article 22
- Visualyse Coordinate: to support the coordination of satellite Earth Stations

We also undertake consultancy work including studies and representation at international meetings such as CEPT and ITU-R.

We have used Monte Carlo methods in studies for our clients and algorithms we have developed including for ITU-R Recommendations.

If you have any questions or would like more information please do not hesitate to contact us at:

Email: info@transfinite.com